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It is strange that every subjective feeling so long as it remains within itself can neither be localised nor determined. We know nothing whatever of the brain-motion that thinks a certain idea. We can fairly assume that every idea is in its objective existence a peculiar kind of brain motion taking place in a particular part of the brain, but we are not conscious of the brain-motion as a special and localised motion. We are quite unable to tell the difference that we must suppose to exist between the forms of the brain-structures or combinations of brain-structures and their motions when we think say for instances of virtue and of vice. We are conscious only of the idea and not of their objective correlates.

Whatever we know of our body, we know only through sensation; i. e., by the same means by which we know of other things. Our body is to us, and is represented with the assistance of the senses, as an object in the objective world. As such it is localised and all its relations and activities are determined. Whatever subjective feeling we have concerning any state of ourselves, remains indistinct until with the help of the senses it is made an object to our observation. Who has not as yet made the experience that he was unable to localise a toothache. The pain itself gives no information either as to its nature and cause or as to the seat of the suffering. The pain itself is purely subjective. All the objective facts have to be localised with the assistance of the senses. The suspected regions must be made the object of experiments and if any irritation of a certain spot increases the ache, it will be assumed to be the seat of the pain. And even then how often is a patient mistaken not only almost always as to the nature but often also as to the seat of the pain.

These facts appear strange, but they cease to be strange, when we consider that the nature of subjectivity is feeling. Subjectivity can as little become directly conscious of its own objectivity as an eye can look at itself. However, an eye can look at its image in the mirror. So the complex of subjective existence, which is through the interaction of an organism united in what we call a soul, can and does turn the channels of its own senses back upon itself and thus forms an opinion concerning its own objectivity. Man's knowledge of his own objective existence is not due to any internal and direct perception of self, but solely to the same experience through which he receives information concerning the rest of the world.

P. C.

A REPLY TO A CRITIC.

WITH A DISCUSSION OF NECESSARY TRUTHS.

To the Editor of The Monist:

I hope it is not a breach of etiquette to ask you to forward to your reviewer the following remarks in reply to his criticism of my work (*The Foundations of Geometry*, reviewed in Vol. II, No. 1, of *The Monist*). If he is good enough to review my second book also, I think they will clear up some misunderstandings.

Your reviewer commences with some general remarks, against which I have

nothing to say. He then proceeds to consider my "requirements for a logical definition." Here he seems to find a difficulty—which may be due to my not having expressed myself clearly. If so I hope he will read what I say on the same subject in my *Essay on Reasoning*, which I cannot believe he will find "indefinite" or not well "issuable." But indeed I cannot see where his difficulty comes in with my old statement of the case. I state perfectly clearly that requirements (3) and (4) are not *logically* necessary for a definition, but are only required if that definition is intended to give a *particular meaning* to the word. He tries to reduce my argument *ad absurdum* by giving a definition of "troft." But so far from being absurd his definition is perfectly good. According to it "troft" would include in its denotation all our percepts and concepts. When however he goes on to say "... These significant names must be so used that the intellectual sensibility shall be excited to perceive that which is intended to be defined," I differ from him entirely. This is only required for a *description*, not for a *definition* (see *Essay on Reasoning*, p. 53).

Your reviewer's only solid objection to my "requirements" seems to be that the fourth includes all the rest. This is only true if the term proposed for definition has an import which has already been determined; but even in such a case it is better to consider the requirements separately, as I have given them. For the force of objections under the different headings varies enormously. An objection under heading (1), if established, would be fatal to any definition whatever. One under heading (2) so far from being fatal would only be a suggestion for the improvement of the definition. Objections under either of the headings (3) and (4) would only be to the effect that the term as defined meant something different from what it was desired that it should mean. It is however convenient to consider (3) and (4) separately as it would generally be possible to decide (3) at once, whereas if a doubt were raised under heading (4) it might lead to a prolonged discussion before it could be laid. I do not however pretend that the "requirements" are laid down in my *Foundations of Geometry* in the best possible form. Indeed I have altered the form in my second essay. There is moreover one requirement for a logical definition which is not included in my heading (1) in the *Foundations of Geometry*, though it is included in (4). This defect is remedied in the *Essay on Reasoning* (p. 55). It is curious that your reviewer should have missed this point, as it is the very one on which he attacks my definition of "direction." It is that the assertions in a definition must not be *independent* of the meaning of the term defined. If they were, the assertion would be equally true (or false) whatever meaning the term might have. The *import* of the term would therefore be unlimited. In the case of explicit definitions a similar error is called *circulus in definiendo*.

When your reviewer goes on to attack my definition of "direction" why does he change his front all at once, and disregard all the considerations he has just been discussing? Why does he not apply my, or his own, requirements for a definition to the case in point? The criticism he actually does put forward will not bear a moment's investigation. If my definition is "circular," the assertion must be equally

true whatever meaning is ascribed to the term. Well, then, let us try the effect of giving to it the meaning we ordinarily ascribe to "cheese." Is it equally true that "a cheese may be conceived to be indicated by naming two points, as the cheese from one to the other"? Clearly not. But not only does this one assertion out of my definition exclude the import of "cheese" from the meaning of "direction," but, more particularly, it distinguishes between the "three distinct but closely associated notions" which your reviewer quite rightly says "become confused in thought and expression unless the most solicitous care is taken to distinguish them." This is exactly the care which I *have* taken, by framing my definition.

I need not say much about the rest of the criticism. Your reviewer's remarks on my definition of "angle" are simply due to the fact that he has not read the definition carefully, and probably has not read the note on the top of page 36 at all. It may make it clearer to him if I point out that if "we imagine a northeast-southwest line cutting an east-west line," we imagine *four* different directions and therefore $4 \cdot 3 / 1 \cdot 2 = 6$ angles. Two of these are the straight angles between the opposite directions of each of the two lines. The other four are what Euclid calls "the angles between the lines." As an angle, according to my definition, has no local habitation in space, it is, *prima facie*, meaningless to talk of the "right hand upper angle." But if this is only an abbreviation for "the angle between the directions upwards and to the right," then "the right hand upper angle" means the same as it would in Euclid.

With the remarks about the nature of the challenge I have thrown down I heartily agree. May I however suggest that I have a right to expect that criticism should be, not only "competent and candid," but careful? It is a difficult subject, and I at least am not always able to express myself in such a way that my meaning cannot be misunderstood by any one. I think if your reviewer looks at what I have said again, with the aid of what I say further in my *Essay on Reasoning*, he will see that his criticisms have really originated in misunderstandings, and perhaps he will alter his judgment that I have "come short of the high result to which I aspired."*

But my chief object in writing to you to-day is to bring specially to your notice my ideas on the nature of so-called "necessary truths." I am not quite clear how far you will find my views harmonise with your own. To a great extent I am inclined to think they are simply a further analysis of the views you express in *The Monist*

* The reviewer of Mr. Dixon's book has read these remarks on his criticism (*The Monist*, Vol. II, No. 1, p. 126) and has given them what seems to him full consideration. He confesses that he misunderstood what Mr. Dixon meant by "a direction." (See the article "Logic as Relation-Lore" to be published in a subsequent number.) In regard to the requirements for a logical definition he must still abide by his former opinion. The need of a definition arises either from the inaccuracy in the application of a term or from a supposed lack of knowledge as to its signification. Hence to use the term itself in its own definition is to import into the definition the same vagueness or ignorance which it is the very office of a definition to correct. When Mr. Dixon says that it is requisite for a logical definition that the defining assertions "must not be *independent* of the meaning of the term defined," what is that but to say that the same must be *dependent* upon that meaning? which, unless the reviewer again misunderstands the author, is to say that we must understand the meaning of the term before we can understand the definition. ρσλ.

and in your *Fundamental Problems*. I will briefly sketch my own ideas and you can then judge whether they are yours also or not.

In my *Essay on Reasoning* I classify assertions as Truisms (assertions whose truth depends solely on the definitions of their terms) and Real Assertions, which convey some real subjective or objective information. I show that the validity of all purely formal knowledge depends on the fact that it is deduced from definitions alone, which are laid down *arbitrarily* and that the supposed peculiar certainty of the theorems of pure mathematics is merely due to the fact that they are all truisms. Thus, I think it a misnomer to call such theorems "necessary" truths. It would be nearer the mark to call them "arbitrary" truths.

There is no *necessity* whatever about the theorem "twice two is four." "Two" is defined as $1 + 1$; "twice," as the operation of adding a thing to itself. It follows from this that "twice two" is $1 + 1 + 1 + 1$; and this, by *definition*, is "four." If "four" were defined as $1 + 1 + 1$, (and there is no "necessary" reason why it should not be,) then "twice two" would *not* be "four." The assertion "twice two is four" conveys no real information whatever—at best it could only tell us what one of its terms meant if they had not all been previously defined.

I cannot insist too strongly on the importance of a proper understanding and use of logical definition. If you desire to know whether a given assertion is true or false, *a priori* or *a posteriori*, the first step in the investigation must be to find out how its terms were defined. If it turns out that the truth (or falsehood) of the assertion can be formally deduced from these definitions, then the assertion is a truism (or contradiction in terms): in either case it can give no real information, and even if true cannot be a "necessary" truth. Only if the definitions of the terms are both independent and consistent is it open to discussion how we might come to a knowledge of the fact it expresses.

I may briefly indicate here how I think the problem ought to be attacked. "Objective facts" can only be established by induction. I do not mean by that term necessarily the process described by Mill, but some similar process, based ultimately on *inductio per enumerationem simplicem*. Now no such process can ever lead to a necessary truth. The most fundamental and certain induction which can be made, that which induces us to believe in the objectivity of our environment, does not lead to a "necessary truth"; and much less can any other induction based upon this one do so. "Objective facts" then may be established with greater or less probability, but can never be *necessarily* true. But all inductions are based on our perceptions, that is ultimately on our subjective sensations. And a man can, nay, must be, absolutely certain of the reality of his own sensations though he cannot be certain of the interpretations he puts upon them. If I have a toothache I cannot be absolutely certain that I have a tooth, but, at least while the pain lasts, I am *absolutely* certain that I have an ache. And so of any subjective sensation.

I can similarly be absolutely certain that I entertain a given concept, while that concept is before my mind; though of course it is possible that if I assert the pos-

session of that concept I may do so in language which may be misunderstood by the person I am addressing. If then a man has certain concepts which he can call up at will, the reality of those concepts, *qua* concepts, is to him a *necessary truth*. He may lay down such necessary truths as axioms, and by their aid he may give real subjective import to a symbolic argument, and so obtain new and complicated assertions which are also to him necessary truths. This is what I do in my subjective theory of geometry. That theory might be regarded as purely symbolic—the axioms might have been left out, and all its conclusions looked upon as mere truisms. The conclusions of geometry of four or more independent directions can perhaps *only* be regarded as truisms. But by the aid of the axioms, geometry of two and three independent directions can be given real subjective import, and its conclusions therefore regarded as necessary truths, as long as they are only taken subjectively. They may further be applied objectively by the aid of objective facts established by induction, but in this case their validity is no greater than that of the primary facts, the counterparts of the subjective axioms, which are employed to give the theory objective import.

I confess I have not studied Kant sufficiently to say that his views differ materially from mine, though I always thought they did until I read your interpretations of them. Perhaps I misunderstood the sense in which Kant used the term *a priori*. The term has been used in so many different senses that I prefer myself to drop it altogether. If it merely refers to priority in time there can be no practical doubt that, whether in the case of the human race or of an individual thinker, a large amount of sense-experience must have preceded even so simple an *a priori* judgment as "twice two is four." If the term merely refers to priority in logical validity it seems to me better to say that "such and such assertions are not dependent upon experience." But Kant says of the assertion " $7 + 5 = 12$ " that it is not only "*a priori*" but "synthetic." By the latter term he means that its truth was *not* deduced from definitions alone, and that the assertion therefore conveys real information. In this I believe he was wrong, and though he afterwards declares that "all knowledge *a priori* is empty and cannot give information about things," unless the true nature of *a priori* knowledge is made more clear, people will inevitably continue to believe the contrary—and to believe moreover that Kant taught so.

Any language which seems to imply that there is some dread necessity about mathematical truths—that they could not be otherwise if they would—is very misleading. Of course it is necessarily true that *if* you have seven objects and add five more to them you will have in all twelve objects. But the whole objective difficulty is begged by the supposition. "Much virtue in it!"

As I understand it the essence of the "laws" of pure mathematics is that they are verbal, that is they are only abbreviated expressions of the results of certain verbal processes. If the processes are repeated and the results similarly expressed, the results must always be the same. Our reason cannot "inform us about the form of existence" unless it is first given, as the *data* or facts which correspond to the defi-

nitions of our symbolic arguments. It is only because our reasoning faculties are limited that symbolic arguments are necessary at all—that it is not evident to us at once that the conclusions of the most intricate mathematical calculations are given to us along with the *data*. Given the data, then in all possible worlds the conclusions must indeed follow, but only because they really are already *in* the data which were given.

It may be that you will not only agree with all I have said, but have already said much of it yourself. But there are some passages in your *Fundamental Problems* which seem to imply otherwise. I think the great objection I have to urge against Kant, and also perhaps against you, is that you do not distinguish as clearly as I could wish between symbolic argument and real, though subjective, knowledge. And the only way to distinguish between them is by inquiring into the definitions of the terms.

For example, on p. 165 of *Fundamental Problems* you say that to four-dimensional beings Kepler's third law "would most probably appear as 'the cubes of their times of revolution being proportional to their mean distances to the fourth power.'"

Now what sort of assertion do you take Kepler's law to be? Originally it was a purely empirical law obtained by pure induction. If the four-dimensional people obtained their law the same way why should the result appear different to them? Or do you conceive the law to be deduced from Newton's theory of gravitation? But even so the law of the inverse square was obtained empirically. If you think that law can be explained (as the analogous law for the distribution of light can) by the supposition that the integral of the force over all points at a given distance from the origin is constant, still this supposition is purely gratuitous unless established by induction from experience. If you grant any one of these suppositions you can by symbolic argument obtain the law corresponding to Kepler's for a four-dimensional space. But I may mention that in no case does the result you anticipate come out. On the first two suppositions the law would be unaltered. On the last supposition the law of gravity would be changed to the inverse cube; but after that the solution of the problem has nothing to do with four dimensions—it is a two-dimensional problem only. The result is that in general planets could not move in closed orbits at all. They might conceivably revolve in circles, but such a condition would be unstable, and if it obtained their periodic times would vary as the squares of their distances.

Again you say (p. 74) "the doctrine of the 'conservation of matter and energy,' although it has been discovered with the assistance of experience, can be proved in its full scope by pure reason alone." I should very much like to see your proof (which I cannot find in *Fundamental Problems*). How do you define the terms of the doctrine? Do you deduce the proof from these definitions—that is do you make it a truism? Or do you base it upon subjective axioms as I do my geometry? Or if you base it on objective facts, how do you prove those facts by pure reason alone?

And if it is purely a subjective proof, how can you say the doctrine is proved "in its full scope"? Surely objective applications come within its scope?

It would not be fair in me to ask you to publish my reply to your reviewer's criticisms, though if that reply is justified the criticism must have done the prospects of my book some injury, seeing from what a quarter it comes. But I hope you will see your way to publishing the latter part of this letter in *The Monist*, together with your reply to it, if you think it worthy of such a distinction.

I have just come across, in this month's *Nineteenth Century*, another remarkable instance of reasoning which seems to be rendered entirely nugatory by the want of proper definitions. It is asserted that conceptual thought is impossible without language. At first sight this would certainly appear to be a real assertion. It follows from it that since dogs have no language they have no "conceptual thought." But it may be plainly shown that dogs do entertain "general notions," which in ordinary English would be included under the head of "conceptual thought." The apparent contradiction is however explained when it appears that the author distinguishes general notions as "concepts" or "recepts," according as they *are* or *are not named*. This being his definition of "conceptual thought" as opposed to other thought, it appears that the assertion is only a truism after all, and conveys no real information whatever. To discuss it further is then mere waste of time. The author of the assertion doubtless *wished* it to convey some information, but he did not attend to his definitions and so failed to attain his object.

EDWARD T. DIXON.

MATHEMATICS A DESCRIPTION OF OPERATIONS WITH PURE FORMS.

IN REPLY TO MR. EDWARD DIXON.

It is true, as Mr. Dixon says, that "Any language which seems to imply that there is some dread necessity about mathematical truths is very misleading." But to say, as Mr. Dixon does in another passage, that the truisms of mathematics are arbitrary truths, is more misleading still. The theorems of the formal sciences are not "assertions whose truth depends solely on the definition of their terms." They are "real assertions which convey some real subjective or objective information."

Mr. Dixon objects to Kant's assertion that $7 + 5 = 12$ is not only *a priori* but also synthetic. He declares, in contradistinction to Kant, that it is deduced from definitions alone; that therefore it is empty, and cannot give any information about things. This latter proposition, which is a phrase of Kant's, appears in this context as an inconsistency of Kant's. And it would be an inconsistency, if it had to be understood in the sense in which Mr. Dixon quotes it. We construe Kant's phrase that "the *a priori* is empty, and cannot give information about things," in a different way. We think that Kant intends to say that the *a priori* imparts real information concerning relations and forms; but that it does not impart real information con-